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The Authorship Dilemma: Alphabetical or Contribution?

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Abstract Scientific communities have adopted different conventions for ordering authors on publications. Are these choices inconsequential, or do they have significant influence on individual authors and research communities at large? What are the trade-offs of using one convention over another?

In order to investigate these questions, we formulate a basic game theoretic model, which already illustrates interesting phenomena that can occur in more realistic settings. We find that contribution-based ordering leads to a denser collaboration network and a greater number of publications, while alphabetical ordering can improve research quality. Contrary to the assumption that free riding is a weakness of the alphabetical ordering scheme, this phenomenon can occur under any contribution scheme, and the worst case occurs under contribution-based ordering. Finally, we show how authors working on multiple projects can cooperate to attain optimal research quality and eliminate free riding given either contribution scheme.

Keywords game theory · coalitional games · academic collaboration · credit allocation · resource allocation

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1 Introduction

Resource allocation is a central problem in artificial intelligence and more generally, economic activity is fundamentally about resource allocation. Without loss of generality, every decision – and thus every computation – can be viewed as a resource allocation instance [23]. Research has been recognized as an economic activity that is crucial for the long term well-being of society, and as a result, developed countries allocate a significant percentage of their resources to basic and applied research activities. The credit allocation schemes should incentivize scientific communities to operate at their best; however, decisions regarding the distribution of research resources are sometimes made in an adhoc manner, with little theoretical or empirical justification of their long-term performance. In this paper, we investigate one of the core problems in this domain, namely, the allocation of credit for scientific work.

The allocation of scientific credit influences funding decisions, as well as tenure, promotions, and awards. Given the critical role that credit allocation plays in academia, it is surprising how little is known of the effects of name-ordering conventions. What influence do ordering schemes have on individual authors? Do they have any global effect on the research communities where they are applied? What can authors do to overcome the limitations imposed by credit allocation schemes?

Our goal in this paper is to provide a framework for studying the effects of author ordering schemes and address these questions. While many ordering conventions are possible, the prominent ones in academic communities are to list authors *by contribution*, that is, in descending order of their contribution to the paper, or *alphabetically*, in lexicographical order of their last names.

Lake [17] shows that listing authors alphabetically gives rise to the *Matthew Effect*, whereby readers are likely to assume that the more established authors deserve more credit. Furthermore, alphabetical ordering benefits those whose last names start with letters that occur earlier in the alphabet [9, 10, 22]. Tenure at highly ranked schools, fellowship, and to some extent even Nobel Prize winnings are correlated with surname initials [10]. Indeed, the American Psychological Association [1] mandates ordering authors by their contribution:

> *"name of the principal contributor should appear first, with subsequent names in order of decreasing contribution."*

On the other hand, other major disciplines such as mathematics, theoretical computer science, and some branches of economics have a long tradition of relying on alphabetical ordering. The American Mathematical Society [2] states:

> *"Determining which person contributed which ideas is often meaningless because the ideas grow from complex discussions among all partners... mathematicians traditionally list authors on joint papers in alphabetical order."*

Moreover, some studies indicate that alphabetical ordering can result in improved research quality. Brown *et al.* [5] found that alphabetical ordering is positively correlated with research quality in the marketing literature, while Laband and Tollison [16] observed a similar trend in economics research, where two-author papers that are alphabetized are cited more frequently. In addition, Joseph *et al.* [14] set up a two agent simulation, and found that the tendency towards alphabetical author ordering increases as acceptance rates decrease, and that for a fixed acceptance rate, papers whose authors are listed alphabetically tend to be of higher quality.

While no scheme is optimal, it is important to first understand the strengths and weaknesses of each, such that each research community can implement the one most useful for its types of collaborations. Our study focuses on projects completed by one or two parties, which is consistent with the rest of the literature on network formation games and collaboration models. One or two-authored papers represent a substantial fraction of the literature in some fields, including mathematics, physics, and economics. Laband and Tollison [16] show that, based on number of citations, two authors appear to represent the optimal team size in economics. Newman [19] finds that:

"[...] purely theoretical papers appear to be typically the work of two scientists, with high-energy theory and computer science showing averages of 1:99 and 2:22."

We collected data from the proceedings of major artificial intelligence conferences over the years 2013–2015 and found that approximately 30% of the published papers are completed by one or two researchers (see Table 1). These numbers reinforce the study of one and two-authored papers as an important first step towards understanding the impact of credit allocation schemes.

1.1 Our Contributions

We formulate a game theoretic model of collaboration, which allows the investigation of credit allocation schemes and illustrates several important phenomena. Our formulation offers a compelling explanation for the phenomenon that alphabetical ordering can lead to improved research quality in some communities. In particular, alphabetical ordering encourages collaborators to match each other's efforts: when one of the authors invests a lot of effort into a project, it is a best response for the co-author(s) to also invest high effort, since matching efforts leads to higher utility.

On the other hand, we find that contribution-based ordering can result in the completion of more research projects and a denser social network. The latter phenomenon has been observed through empirical data analysis by Newman [20].

When it comes to free riding, at first sight, the issue appears particularly problematic for alphabetical ordering. However, upon further examination, it becomes apparent that both schemes are subject to some degree of free riding and we show that the worst case occurs under contribution-based ordering.

As with every theoretical study of social behavior, there is a question of what properties observed in theory apply in practice. We argue that our model

Conference	1	2	3	4	5	6	7	8	9
IJCAI 15	5,96	34,86	29,81	17,88	6,42	2,75			
IJCAI 13	5,69	29,27	30,31	20,46	10,10	3,36			
AAAI 15	7,51	25,26	25,05	22,96	12,52	4,17	1,67	1,04	
AAAI 14	3,34	23,62	30,78	23,62	10,97	5,72	1,19	0,71	
AAAI 13	5,44	29,20	34,65	14,35	8,41	5,94	1,48	0,49	
AAMAS 15	0,97	28,57	37,01	18,18	10,38	3,57	1,94		
AAMAS 14	5,67	26,81	35,96	19,24	8,83	2,83	0,94		
AAMAS 13	8,26	29,52	31,49	20,07	7,87	2,75	1,18	0,39	
UAI 15	1,01	51,51	26,26	12,12	7,07	1,01			
UAI 14	8,33	34,37	30,20	19,79	5,20	2,08			
UAI 13	2,73	39,72	27,39	23,28	6,84				
COLT 15	10	28,57	34,28	22,85	4,28				
COLT 14	7,69	34,61	40,38	11,53	5,76				
COLT 13	19,14	29,78	31,91	10,63	6,38	2,12			
NIPS 15	2,97	28,28	34,49	21,33	8,43	2,97	1,24	0,24	
NIPS 14	2,94	28,67	35,04	22,54	6,12	3,18	0,98	0,49	
NIPS 13	5,11	28,40	32,95	23,29	8,23	0,85	0,85		
ICML 15	1,85	30	33,33	22,22	9,25	1,85	0,37	0,74	0,37
ICML 14	2,58	30,96	35,48	21,61	7,09	1,61	0,32	0,32	
ICML 13	6,00	26,50	33,21	23,67	8,48	1,76	0,35		

Table 1 Major conferences in artificial intelligence with percentage of papers written by $1, 2, \ldots, 9$ authors, respectively, during the years 2013–2015. Approximately 30% of the published papers are completed by at most two authors.

offers a simple and intuitive explanation for several important phenomena observed empirically; moreover, it makes predictions that can be verified experimentally. The study of extended models, with additional properties such as diminishing returns, larger coalitions, divisible budgets, different distributions of individual effort, and dynamic aspects of social networks remains a subject for future work.

1.2 Related Work

In a recent paper, Kleinberg and Oren [15] investigate a related question: Why do some academic communities over-emphasize the importance of highly technical problems? The authors use a noncooperative model for the allocation of scientific credit and their main finding is that research communities may have to over-reward their key scientific challenges, to ensure that such problems are solved in a Nash equilibrium.

The academic game is a non-transferable utility game with overlapping coalitions and is related to several types of coalitional games, such as threshold task games (Chalkiadakis *et al* [6]), coalitional skill games (Bachrach and Rosenchein [4]), and weighted voting games (Elkind *et al* [11]).

There are several co-authorship models in the economics and computer science literature. de Clippel *et al* ([7]) study the division of a homogeneous divisible good when every agent reports an evaluation of the others' contribution, and establish the existence of a unique impartial and consensual mechanism for three agents. Jackson and Wolinsky ([13]) introduce a co-author model with network externalities, where each agent has a unit of time and can divide it among different collaborations, and study the structure of the networks in the equilibrium. Anshelevich and Hoefer [3] study the price of anarchy for contribution games on networks with concave and convex reward functions.

2 Model

The *academic game* studied here is a collaboration model defined as a network formation game. This simple, yet expressive formulation captures the fundamental aspect of collaboration – namely that multiple individuals can do more than one – and enables the investigation of network effects due to name ordering schemes.

Let $N = \{1, \ldots, n\}$ be a set of agents. Each agent i has a budget of weight w_i, consisting of a set of coins $C_i = \{c_{i,1}, \ldots, c_{i,n_i}\}$. Every coin $c_{i,j}$ has a positive weight $w_{i,j}$, and $\sum_{j=1}^{n} w_{i,j} = w_i$, $\forall i \in N$. The agents can work alone or in pairs to solve different projects. A project of weight w can be solved either by:

- one agent who invests a coin of weight w to the project, or
- two agents, each of which contributes with one coin, such that the sum of the two coins is w.

An agent can participate in multiple projects simultaneously by investing a different coin in each project. The same pair of agents can solve multiple projects together, and each coin can only be used once.

2.1 Reward Function

Let T denote a set of project weights, such that for every available project weight $w \in T$, there exist infinitely many projects of this weight. Let $\mathcal{F} : \mathbb{R}^+ \to \mathbb{R}^+$ be a reward function; solving a project of weight w gives a reward $\mathcal{F}(w)$, which is divided among the participants on the project.

In this work we study games with a very general class of rewards, namely convex homogenous functions. Homogeneous valuations are often used in economic theory and have been widely studied in resource allocation domains (for example, in the setting of multiple goods, the class of homogeneous valuations contains well-studied instances such as additive linear and Leontief, to Constant Elasticity of Substitution and Cobb-Douglas ([18,8])). In our model, convexity translates to the property that greater effort leads to greater reward; the rate at which the increase is observed varies depending on the degree of the homogeneous function.

Formally, a function $\mathcal{F}$ is *homogeneous* if there exists degree $d > 0$ such that $\mathcal{F}(t \cdot x) = t^d \cdot \mathcal{F}(x)$, for every x and $t > 0$. Since we study a one-dimensional setting with convex reward, we have: $\mathcal{F}(x) = \alpha \cdot x^d$, where $\alpha = \mathcal{F}(1)$ is a constant and $d > 1$.

Finally, in each academic community there is a general perception of the significance of being the first or second author on a paper. Without prior knowledge about the specific paper or its authors, the relative contribution of each author on a two-authored paper can be represented by a fixed *contribution vector* $[\phi, 1-\phi]$, where $\frac{1}{2} \leq \phi < 1$. That is, the community assumes that the contribution of the first and second author are ϕ% and $(1-\phi)$%, respectively. Throughout the paper, we refer to the case where $\phi = \frac{1}{2} = 1-\phi$ as *alphabetical ordering*, and to the cases where $\phi > \frac{1}{2} > 1-\phi$ as *contribution-based ordering.*

Most of our results are concerned with indivisible budgets. To this end, we note that while time is divisible, other resources are not, and so indivisible budgets convey whether an author is committed to a project.

2.2 Coalition Structures

Next, we define coalition structures and utility in academic games.

Definition 1 Given an academic game, a *coalition structure* is a partition of the set of all coins, such that every coin $c_{i,j}$ of agent i is either a singleton project, or is paired with a coin $c_{k,l}$ belonging to another agent $k \in N \setminus \{i\}$.

Definition 2 Given an academic game and a coalition structure CS, let CS_i be the set of projects that agent i contributes to, $\forall i \in N$. The utility of i is:

$$u_i(CS) = \sum_{P_j \in CS_i} v_i(P_j),$$

where $\{P_1, \ldots, P_m\}$ is the set of projects solved under CS, $w(P_j)$ is the weight of project P_j, and

$$v_i(P_j) = \begin{cases} w(P_j)^d & \text{if } i \text{ completes } P_j \text{ alone} \\ \phi \cdot w(P_j)^d & \text{if } i \text{ is first author on } P_j \\ (1-\phi) \cdot w(P_j)^d & \text{if } i \text{ is second author on } P_j \end{cases}$$

Next we give an example of an academic game.

Example 1 Consider an academic game with two agents, where $\alpha = 1$, $d = 2$, agent 1 has the set of coins $C_1 = \{c_{1,1}, c_{1,2}\}$, agent 2 has the set $C_2 = \{c_{2,1}\}$, and the weights of the coins are: $w_{1,1} = 3, w_{1,2} = 1$, $w_{2,1} = 2$. The possible coalition structures are: $CS_1 = (\{c_{1,1}\}, \{c_{1,2}\}, \{c_{2,1}\})$, $CS_2 = (\{c_{1,1}, c_{2,1}\}, \{c_{1,2}\})$, and $CS_3 = (\{c_{2,1}, c_{1,2}\}, \{c_{1,1}\})$, where for each project, the coins are listed by decreasing size. The utilities of the agents are as follows:

- CS_1: $u_1(CS_1) = \alpha \cdot w_{1,1}^d + \alpha \cdot w_{1,2}^d = 1 \cdot 3^2 + 1 \cdot 1^2 = 10$ and $u_2(CS_1 b) = \alpha \cdot w_{2,1}^d = 1 \cdot 2^2 = 4$.

- CS_2: $u_1(CS_2) = \phi \cdot \alpha \cdot (w_{1,1} + w_{2,1})^d + \alpha \cdot w_{1,2}^d = \phi \cdot 5^2 + 1^2 = 25\phi + 1$ and $u_2(CS_2) = (1-\phi) \cdot \alpha \cdot (w_{1,1} + \cdot w_{2,1})^d = 25(1-\phi)$
- CS_3: $u_1(CS_3) = (1-\phi) \cdot \alpha \cdot (w_{2,1} + w_{1,2})^d + \alpha \cdot w_{1,1}^d = 9(1-\phi) + 9$ and $u_2(CS_3) = \phi \cdot \alpha \cdot (w_{2,1} + w_{1,2})^d = 9\phi$.

3 Indivisible Budgets

We first study the setting of indivisible budgets, where each agent owns a single coin, corresponding to the scenario where every agent is involved in a single project. This setting is sufficient to differentiate between alphabetical and contribution ordering and highlights an interesting effect. Namely, there exist natural settings in which alphabetical ordering encourages agents to match each others' efforts, and as a result, it leads to the completion of larger projects.

First, we introduce pairwise stability, the standard solution concept in network formation games [12]. A coalition structure CS is *pairwise stable* if:

- For all $i \in N$, $u_i(CS) \geq \alpha \cdot w_i^d$. That is, i cannot improve by allocating his coin to a singleton project.
- For all $i, j \in N$, with $w_i \geq w_j$, either $u_i(CS) \geq \phi \cdot \alpha \cdot (w_i + w_j)^d$ or $u_j(CS) \geq (1-\phi) \cdot \alpha \cdot (w_i + w_j)^d$. That is, i and j cannot deviate by forming a joint project.

Furthermore, we note that the alphabetical ordering scheme guarantees the existence of a pairwise stable coalition structure. In contrast, there exist instance where no pairwise stable coalition exists under contribution-based ordering. See Appendix for details.

3.1 Research Quality

We show that alphabetical ordering can result in higher research quality than is possible under some contribution-based scheme. Since agents can work either by themselves or in pairs, the most difficult project that a set of agents can solve results from the combined efforts of its two strongest agents. We call a project of this difficulty a *hard project.* The same coalition of agents can solve multiple hard projects, by having each agent invest different coins to the different projects.

We begin by considering identical agents.

Proposition 1 *Consider an academic game with identical agents and indivisible budgets. Then every pairwise stable coalition structure solves the maximum number of* hard projects *whenever the credit to the first author is in the range:* $\phi \in \left(\frac{1}{2^d}, \frac{2^d-1}{2^d}\right)$.

Proof When the agents are identical, a project is *hard* if solved by two agents. Without loss of generality, we can assume that each agent has a budget of size 1. In order for the maximum number of hard projects to be solved in every pairwise stable equilibrium, it should be the case that two singleton agents can strictly improve their utility by working on a joint project. The conditions for the first and second author, respectively, are: $\phi \cdot \alpha \cdot 2^d > \alpha \cdot 1^d$ and $(1-\phi) \cdot \alpha \cdot 2^d > \alpha \cdot 1^d$, or equivalently, $\phi \in \left(\frac{1}{2^d}, \frac{2^d-1}{2^d}\right)$.

Note that the maximum number of hard projects is always solved under alphabetical ordering ($\phi = \frac{1}{2}$).

We now consider a game with two agent types, *heavy agents* and *light agents*; the weights are normalized such that the heavy agents invest coins of weight 1 and the light agents invest coins of weight $\lambda \in (0,1)$. A contribution scheme can encourage same-layer collaborations (resulting in the completion of the maximum number of hard projects), or cross-layer collaborations, or simply discourage collaboration (by giving very little credit to second authors, for example).

Theorem 1 *Consider an academic game with indivisible budgets and two types of agents, light and heavy. Then every pairwise stable coalition structure has:*

(1) *Only same-layer collaborations when:*

$$\frac{(1+\lambda)^d}{2^d+(1+\lambda)^d} < \phi < \min\left\{\frac{2^d-1}{2^d}, \frac{1}{(1+\lambda)^d}, \frac{2^d}{2^d+(1+\lambda)^d}\right\}$$

(2) *Only cross-layer collaborations when:*

$$\max\left\{\frac{2^d-1}{2^d}, \frac{1}{(1+\lambda)^d}, \frac{2^d}{2^d+(1+\lambda)^d}\right\} < \phi < 1-\left(\frac{\lambda}{1+\lambda}\right)^d$$

(3) *No collaboration when:*

$$\frac{2^d-1}{2^d} < \phi < \frac{1}{(1+\lambda)^d} \text{ or } \phi > \max\left\{\frac{2^d-1}{2^d}, 1-\left(\frac{\lambda}{1+\lambda}\right)^d\right\}$$

Proof We prove Case 1 (the other cases are similar). In order for every pairwise stable coalition structure to solve the maximum number of same-layer collaborations, it should be the case that whenever a coalition structure contains:

(a) *Two identical singleton projects:* the two agents can improve by deviating to a pair. That is, $(1-\phi) \cdot \alpha \cdot 2^d > \alpha \cdot 1^d$ and $(1-\phi) \cdot \alpha \cdot (2\lambda)^d > \alpha \cdot \lambda^d$, and so $\phi < \frac{2^d-1}{2^d}$.

(b) *Two cross-layer projects:* then there exists an improving deviation by two agents from the same layer. It is sufficient to require that the two heavy agents involved in the cross layer projects deviate together: $\phi \cdot \alpha \cdot 2^d > \phi \cdot \alpha \cdot (1+\lambda)^d$ and $(1-\phi) \cdot \alpha \cdot 2^d > (1-\phi) \cdot \alpha \cdot (1+\lambda)^d$. Thus $\phi < \frac{2^d}{2^d+(1+\lambda)^d}$.

(c) *One cross-layer project and one heavy singleton project:* the two heavy agents deviate to a pair when $\phi{\cdot}\alpha{\cdot}2^d > \phi{\cdot}\alpha{\cdot}(1{+}\lambda)^d$ and $(1{-}\phi){\cdot}\alpha{\cdot}2^d > \alpha{\cdot}1^d$, thus $\phi < \frac{2^d-1}{2^d}$.

(d) *One cross-layer project and one light singleton project:* the two light agents deviate to a pair when $\phi{\cdot}\alpha{\cdot}(2\lambda)^d > (1{-}\phi){\cdot}\alpha{\cdot}(1{+}\lambda)^d$ and $(1{-}\phi){\cdot}\alpha{\cdot}(2\lambda)^d > \alpha\cdot\lambda^d$. That is: $\frac{(1+\lambda)^d}{(2\lambda)^d+(1+\lambda)^d} < \phi < \frac{2^d-1}{2^d}$.

In addition, we require that no cross-layer project is solved, even when the maximum number of hard projects is completed. That is, a coalition of weight $1+\lambda$ is blocked by a deviation to a singleton by one of the agents, and so $\phi < \frac{1}{(1+\lambda)^d}$ or $\phi > 1 - \left(\frac{\lambda}{1+\lambda}\right)^d$. It follows that:

$$\frac{(1+\lambda)^d}{2^d+(1+\lambda)^d} < \phi < \min\left\{\frac{2^d-1}{2^d}, \frac{1}{(1+\lambda)^d}, \frac{2^d}{2^d+(1+\lambda)^d}\right\}.$$

Observe that by setting $\phi = \frac{1}{2}$, we obtain that alphabetical ordering solves the highest number of hard projects, while ordering by contribution in the range given by Case 2 solves the highest number of intermediate projects (requiring one heavy and one light coin).

3.2 Free Riding

It has been argued that alphabetical ordering is unfair [17], as it gives the same credit to all authors even when they do not contribute equally. However, the fundamental difficulty leading to free riding is that the contribution scheme is fixed, not whether it is alphabetical or contribution-based.

Even when authors are ordering by contribution, members of academic communities have predetermined notions of the proportion of work contributed by each author[1]. Authors cannot choose the contribution vector, as doing so would require changing the perception of the entire community.

We show that the use of a fixed contribution vector necessarily leads to the free riding effect. In addition, the degree of free riding admitted by alphabetical ordering is *not* the worst possible.

Formally, if two agents allocate weights x and y, respectively, to a joint project, then the rewards should be proportional to the effort invested, i.e. $\left(\frac{x}{x+y}\right)(x+y)^d$ and $\left(\frac{y}{x+y}\right)(x+y)^d$, respectively.

Thus, the *fair contribution vector* for this project is uniquely defined as: $\mathcal{C} = \left[\frac{x}{x+y}, \frac{y}{x+y}\right]$. All other contribution vectors result in free riding.

For each agent, the *free riding index* is the (normalized) difference between the perceived contribution and the actual contribution. Recall that $w(P)$ denotes the weight of project P. Given a contribution scheme ϕ, the *free riding*

[1] For instance, while in some communities the second author is assumed to have done moderately less than the first, in others, the contribution of the second author is considered to be negligible compared with that of the first.

index of agent i in a coalition structure CS where he solves project P is:

$$\mathcal{L}_i = \begin{cases} 0 & \text{if } i \text{ completes } P \text{ alone} \\ \frac{\phi \cdot w(P) - w_i}{w(P)} & \text{if } i \text{ is the first author on } P \\ \frac{(1-\phi)w(P) - w_i}{w(P)} & \text{if } i \text{ is the second author on } P \end{cases}$$

Note that the free riding index parallels the price of anarchy (Nisan *et al*, [21]).

We begin by considering identical agents. Since they contribute equally to a project, alphabetical author ordering corresponds to the unique fair contribution vector for their project. In particular, larger values of ϕ result in more free riding for first authors.

Theorem 2 *Consider an academic game with identical agents and indivisible budgets. Then alphabetical ordering is the unique fair contribution vector, while when the credit to the first author, ϕ, is greater than $\frac{2^d-1}{2^d}$, then every pairwise stable coalition structure has a free riding index of $\phi - 1/2$ for at least $n/2 - 1$ of the agents.*

Proof It is immediate that first authors always benefit from this collaboration. The second authors would only participate as long as $(1-\phi) \cdot \alpha \cdot 2^d > \alpha \cdot 1^d$, that is, as long as $\phi < 1 - \frac{1}{2^d}$.

The free riding index of all first authors is $\phi - \frac{1}{2}$.

The free riding index is highest when the contribution vector is steep; the credit to the first author can be as high as $\phi = \frac{2^d-1}{2^d}$ without preventing the second author from collaborating. In this case, the free riding index of all first authors is $\frac{1}{2} - \frac{1}{2^d}$. For example, when $\alpha = 1$ and $d = 2$, the maximum free riding index is 25%. In general, the higher the reward of collaboration, the more free riding can occur.

Corollary 1 *There exist academic games and contribution-based ordering schemes such that in every pairwise stable coalition structure, the free riding index is $\frac{1}{2} - \frac{1}{2^d}$ for half of the agents.*

Next, we find the largest free riding index that can occur under alphabetical ordering.

Proposition 2 *Consider an academic game with heavy and light agents, of weights 1 and λ respectively. Then in every coalition structure that is pairwise stable under alphabetical ordering, all the light agents that collaborate with heavy agents have a free riding index of $\frac{1}{2}(1-\lambda)$.*

Proof In the two agent setting, alphabetical ordering benefits the light agents. In order for the heavy agents to collaborate with light agents, it must be the case that $\frac{1}{2} \cdot \alpha \cdot (1+\lambda)^d > \alpha \cdot 1^d$, or, equivalently, $\lambda > 2^{\frac{1}{d}} - 1$.

The free riding index of the light agents is then $\frac{\frac{1}{2}(1+\lambda)-\lambda}{1+\lambda}$ and the worst case is obtained when $\lambda = \frac{1}{2^{\frac{1}{d}}} - \frac{1}{2}$.

It follows that the largest possible free riding index under alphabetical ordering is no greater than $2^{-\frac{1}{d}} - \frac{1}{2}$. For example, when $d = 2$, the free riding index is $\approx 20\%$.

While in the previous results we showed that the contribution vector can affect as many as half of the agents involved, the same worst case bounds hold for individual agents in arbitrary games. The following result follows by combining Proposition 2 and Corollary 1.

Theorem 3 *Consider an academic game in which the agents have indivisible budgets of arbitrary sizes. Then the highest free riding index of any agent under alphabetical ordering is at most* $\frac{1}{2}^{\frac{1}{d}} - \frac{1}{2}$, *while it can be as high as* $\frac{1}{2} - \frac{1}{2^d}$ *under some contribution-based ordering schemes. Moreover, the highest amount of free-riding that occurs in any project solved under alphabetical ordering is smaller than under contribution-based ordering.*

Proof Note that the bound in Proposition 2 represents the highest free riding index that any agent can incur under alphabetical ordering. The result follow from Proposition 2 and Corollary 1.

To show that the highest amount of free-riding that occurs in any project solved under alphabetical ordering is smaller than under contribution-based ordering, we need to show that $\frac{1}{2}^{\frac{1}{d}} - \frac{1}{2} \leq \frac{1}{2} - \frac{1}{2^d}$, or equivalently, $\frac{1}{2}^{d} + \frac{1}{2}^{\frac{1}{d}} \leq 1$. The inequality can be shown by analyzing the behavior of the following function: $f : [0, \infty) \to \mathbb{R}$, where $f(x) = \frac{1}{2}^{x+1} + \frac{1}{2}^{\frac{1}{x+1}}$. Note that $f(0) = 1$; by looking at the derivatives of f, it can be seen that the function is strictly smaller than 1 for all $x > 0$, and reaches 1 again in the limit of $x \to \infty$.

To conclude, even though at first sight, alphabetical ordering appears to be particularly susceptible to free-riding, further investigation reveals that no contribution scheme is immune to free riding; neither listing authors alphabetically nor listing them in decreasing order of contribution can eliminate this effect. Moreover, the worse case attained using alphabetical ordering is better than that of some contribution based schemes.

4 Discrete Budgets

We now turn our attention to the general model, where each agent has multiple coins, allowing agents to work on multiple projects simultaneously. First, note that the results from the single coin setting carry over to discrete budgets. In addition, we uncover several phenomena that cannot be observed in the indivisible budget setting, such as the following: there exist many games in which the *contribution vector does not matter*, since the agents can perform rotations, by alternating between being first and second author on joint projects. Rotations can allow agents to reach optimal research quality as well as obtain perfect fairness.

Our solution concept is pairwise stability for games with overlapping coalition structures. Given that an agent can be involved in multiple projects simultaneously, it is important that one estimates correctly the reactions from the rest of the agents before agreeing to participate in a deviation. We follow the recent literature on overlapping coalition formation games (Chalkadiakis *et al* [6], Zick *et al* [24]), and study *sensitive reactions* to a deviation. In short, when agent i is involved in a deviation from a coalition structure CS, i can expect that:

- Every non-deviating agent which is hurt by the deviation retaliates and drops all the projects with i. Note that unless i and j agreed to deviate together, an agent j is hurt by the deviation when at least one of j's projects has been discontinued by the deviator(s).
- The unaffected agents are neutral and maintain all of their existing projects with i.

Definition 3 A coalition structure CS is pairwise stable if:

- No agent i can drop some of his existing projects and strictly improve in the new coalition structure, $CS^{'}$
- No two agents i and j, can rearrange the projects among themselves and possibly drop some of the projects with the remaining agents, such that both i and j strictly improve their utility in $CS^{'}$,

where $CS^{'}$ is the resulting coalition structure, in which non-deviators have sensitive reactions to a deviation.

The next example illustrates pairwise stability for discrete budgets.

Example 2 Consider an academic game with three agents, sets of coins: $C_1 = \{c_{1,1}, c_{1,2}\}$, $C_2 = \{c_{2,1}, c_{2,2}\}$, and $C_3 = \{c_{3,1}, c_{3,2}\}$, and the coalition structure $CS = (\{c_{1,1},\ c_{2,1}\},\ \{c_{2,2},\ c_{3,1}\},\ \{c_{1,2},\ c_{3,2}\})$.

If agent 1 deviates by allocating the coin $c_{1,2}$ to a singleton project, then 1 expects that the resulting coalition structure is $CS^{'} = (\{c_{1,1},\ c_{2,1}\},\ \{c_{1,2}\},\ \{c_{3,2}\},\ \{c_{2,2},\ c_{3,1}\})$, since agent 2 is not hurt by the deviation.

On the other hand, if the deviating coalition is $\{1, 2\}$ and the deviation consists of forming the joint project $\{c_{1,1}, c_{2,2}\}$, then 1 and 2 expect the resulting coalition structure is $CS^{''} = (\{c_{1,1},\ c_{2,2}\},\ \{c_{2,1}\},\ \{c_{1,2}\},\ \{c_{3,1}\},\ \{c_{3,2}\})$, since agent 3 is hurt by the deviation and drops *all* the projects with the deviators.

4.1 Rotations

Agents can sometimes overcome the limitations of a fixed contribution scheme. That is, they can simultaneously solve the highest number of hardest projects and eliminate free riding, regardless of the contribution vector. We refer to this phenomenon as *rotations*: agents collaborating on multiple projects agree that one of them is the first author on half of their projects, while the other is first on the remaining projects, regardless of whether this represents their actual contributions.

We first demonstrate this result for budgets with multiple identical coins.

Theorem 4 *There exist academic games with discrete budgets and multiple identical coins such that for every ϕ, the maximum number of hard projects is solved in a pairwise stable equilibrium and no free riding occurs.*

Proof Let $\phi < 1$ and consider a two agent game, such that agent 1 has coins $\{c_{1,1}, c_{1,2}\}$, agent 2 has coins $\{c_{2,1}, c_{2,2}\}$, and all the coins have weight 1. Consider the coalition structure $CS = (C_1, C_2)$, where $C_1 = \{c_{1,1}, c_{2,1}\}$ and $C_2 = \{c_{2,2}, c_{1,2}\}$, such that agent 1 is the first author on project C_1 and agent 2 is the first author on project C_2.

It can be verified that both agents receive the best possible utility, which coincides with the fair allocation given by alphabetical ordering. Moreover, CS is pairwise stable: no agent can gain by investing the coin from their second-author project to a singleton project, since the other agent will retaliate and drop the other project as well.

Rotations can also be used to eliminate free riding when coins are not required to be identical, and projects require the combination of different coin types.

Next we study rotations when there exists a conference tier. That is, the projects have weights drawn from $T = [t, \infty)$, where t is the *conference tier.* Solving a project of weight w gives reward $\mathcal{F}(w)$ if $w \geq t$, and zero otherwise.

Proposition 3 *Let $1 + \lambda$ be the conference tier. Then there are academic games with discrete budgets, where each agent has an equal number of light and heavy coins, of weight 1 and $\lambda \in (0, 1)$, respectively, such that there exist pairwise stable coalition structures with no free riding.*

Proof Let $\phi < 1$ and consider a two agent game, such that agent 1 has coins $\{c_{1,1}, c_{1,2}\}$, agent 2 has coins $\{c_{2,1}, c_{2,2}\}$, and all the coins have weight 1. Consider the coalition structure $CS = (C_1, C_2)$, where $C_1 = \{c_{1,1}, c_{2,1}\}$ and $C_2 = \{c_{2,2}, c_{1,2}\}$, such that agent 1 is the first author on project C_1 and agent 2 is the first author on project C_2. It can be verified that both agents receive the best possible utility, which coincides with the fair allocation given by alphabetical ordering. Moreover, the coalition structure is pairwise stable. None of the agents can gain by investing the coin from their second-author project to a singleton project, since the other agent will retaliate and drop the other project as well.

4.2 Implications for the Social Network

For the next result we illustrate the following phenomenon observed in [20]: when the agents use ordering by contribution, they have more co-authors than when using alphabetical ordering. We consider the setting in which every agent has a budget consisting of heavy coins and light coins. The light coins represent very little effort, such as " cheap talk", but can contribute to improving the quality of a paper. Allowing for cheap talk results in a much higher number of collaborations.

Theorem 5 *Consider an academic game with discrete budgets, where each agent has several heavy and light coins, of sizes* 1 *and* ε*, respectively, such that* $0 < \varepsilon \ll 1$*. Moreover, the conference tier is* 1 *and each agent has more heavy coins than light coins.*

Then whenever $\phi > \max\left(\frac{2^d}{2^d+(1+\varepsilon)^d}, \frac{1}{(1+\varepsilon)^d}\right)$*, every pairwise stable equilibrium solves the maximum number of projects and the average number of collaborators per agent is the highest possible.*

Proof To ensure that every pairwise stable coalition structure solves the maximum number of collaborations, the best investment of a heavy coin should be to pair it with a small coin. That is, an agent prefers being the first author on a project of weight $1+\varepsilon$ instead of either second author on a hard project or the only author on a singleton project. The conditions are: $(1-\phi)\cdot 2^d < \phi\cdot(1+\varepsilon)^d$ and $\phi\cdot(1+\varepsilon)^d > 1$; equivalently, $\phi > \max\left(\frac{1}{(1+\varepsilon)^d}, \frac{2^d}{2^d+(1+\varepsilon)^d}\right)$.

Then in every pairwise stable coalition structure, all the heavy coins are paired with small coins, and the average number of collaborators is maximal. Note that while alphabetical ordering solves the highest number of hard projects, both the number of projects completed and the number of collaborators are twice as low. Finally, the singleton coalition structure solves the same number of projects above the conference tier. However, in this case, the agents have no collaborators, and the quality of the projects is lower compared to the case when cheap talk is allowed.

There exist games in which ordering by contribution allows to simultaneously maximize the number of hard projects and the total number of projects. We illustrate this phenomenon when there exist both an upper and lower bound on the hardness of the rewarded projects.

Corollary 2 *Consider an academic game with discrete budgets, where each agent has several heavy and light coins, of sizes* 1 *and* $\lambda \in (0,1)$*, respectively, the conference tier is* 1*, and the maximum project hardness is* $1+\lambda$*. Each agent has more heavy coins than small coins.*

Then whenever $\phi > \max\left(\frac{2^d}{2^d+(1+\lambda)^d}, \frac{1}{(1+\lambda)^d}\right)$*, every pairwise stable equilibrium solves the maximum number of projects, each of the projects solved is the hardest possible, and the average number of collaborators per agent is the highest possible.*

5 Discussion

Authorship order is a central, yet often overlooked, aspect of academia. We introduced a basic game theoretic model for studying author ordering schemes, which already illustrates interesting phenomena that can occur in richer domains. The model offers a compelling explanation for several real-world phenomena, showing that alphabetical ordering is positively correlated with research quality in some scenarios, while contribution based ordering results in a larger number of projects completed and denser social networks.

Our model makes several predictions on the effects of author ordering schemes, which prompt further theoretical and empirical study. In particular, we show that rotations can be used to overcome the limitations of fixed ordering schemes and that the worst case of free riding occurs under some contribution-based schemes. It would be interesting to empirically evaluate how frequently free riding and rotations appear in practice, as well as their influence on individuals and their communities.

Another important direction for future work is to understand precisely the conditions under which alphabetical ordering is better suited than contribution-based ordering, and vice-versa. In those communities where alphabetical ordering is indeed the closest to optimal scheme, policy changes may be called for to alleviate the effect of alphabetical ordering of unfairly favoring the authors with earlier names in the alphabet. Such changes could include changing the citation styles for alphabetical papers or possibly using random ordering, with a note that the contribution is meant to be weighted equally. Moreover, it would be interesting to study models that capture additional realistic phenomena, such as reward functions with diminishing returns, arbitrary coalition sizes, heterogenous skill sets, and dynamics of social networks that may influence the equilibria reached.

More generally, the following implementation theory question remains open: *Given a scientific community, what is the optimal credit allocation scheme?* This paper is a first step in the direction of understanding this question, which is at the heart of resource allocation in academic research and arguably the long-term development of society. We hope that this work will stimulate further discussion and research on the topic [2].

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[2] The short version of this paper already led to a thought-provoking online discussion in the game theory community [**?**]

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5.1 Appendix

We show that alphabetical author ordering guarantees the existence of a pairwise stable coalition structure; furthermore, it can always be found in polynomial time. In contrast, the use of contribution based ordering can result in there being no pairwise stable solutions.

Theorem 6 *Every academic game with identical agents and indivisible budgets has a pairwise stable coalition structure under deterministic tie-breaking rules.*

Proof Without loss of generality, assume that the deterministic tie-breaking rule lists the agents in the order $[1, \ldots, n]$. If $\phi \geq 1 - \frac{1}{2^d}$, then the singleton coalition structure, $CS = (\{1\}, \ldots, \{n\})$ is pairwise stable, since no agent can improve their utility by being second author on a project. If $\phi < 1 - \frac{1}{2^d}$, there are two cases. If n is even, then $CS^{'} = (\{1, 2\}, \ldots, \{n-1, n\})$ is pairwise stable. If n is odd, then $CS^{''} = (\{2, 3\}, \ldots, \{n-1, n\}, \{1\})$, is pairwise stable, since no agent has an incentive to switch to a singleton, the coalition structure given by $CS^{''} \setminus \{1\}$ is pairwise stable (by case 1), and agent 1 cannot be part of a deviating pair, since no other agent has an incentive to join 1 as a second author.

Theorem 7 *Consider an academic game with different agents and indivisible budgets. Under alphabetical ordering, a pairwise stable coalition structure is guaranteed to exist and can be computed in polynomial time.*

Proof Let $N = \{1, \ldots, n\}$ be the set of agents and without loss of generality, let $w_1 \geq w_2 \geq \ldots \geq w_n$. Start with an empty coalition structure: $CS = \emptyset$. Iteratively, pair whenever possible the two agents with the heaviest weights among the remaining agents. Let $\{k, k+1, \ldots, n\}$ be the remaining set of agents during some iteration. If

$$\frac{1}{2} \cdot \alpha \cdot (w_k + w_{k+1})^d \geq \alpha \cdot w_k^d$$

and

$$\frac{1}{2} \cdot \alpha \cdot (w_k + w_{k+1})^d \geq \alpha \cdot w_{k+1}^d$$

then let $CS \leftarrow CS \cup \{k, k+1\}$, otherwise, $CS \leftarrow CS \cup \{k\}$. We claim that the resulting coalition structure, CS, is pairwise stable. If CS contains coalition $\{1, 2\}$, then agent 1 does not have an incentive to form another pair or move to a singleton, since agent 1's utility in CS, $u_1(CS)$, verifies the following inequalities:

$$u_1(CS) \geq \alpha \cdot w_1^d$$

and

$$u_1(CS) \geq \frac{1}{2} \cdot \alpha \cdot (w_1 + w_j)^d, \forall j \in N \setminus \{1\}$$

Similarly, agent 1 does not deviate if a singleton in CS. Iteratively, whenever the first k agents do not have an incentive to deviate, agent $k+1$ does not have an incentive to deviate either. Thus CS is pairwise stable.

On the other hand, contribution-based ordering does not guarantee the existence of stable coalition structures even under fixed tie-breaking rules.

Proposition 4 *There exist academic games with different agents and indivisible budgets, such that when contribution-based ordering is used, no pairwise stable coalition structure exists.*

Proof Consider a three agent game, with weights 1, $1+\varepsilon$, and $1+2\varepsilon$, respectively, where $\alpha = 1$, $d = 2$, $\varepsilon = 0.8$, and $\phi = 0.8$. It can be easily verified that no coalition structure is stable. The singleton coalition structure is blocked by the agents with weights $\{1, 1+\varepsilon\}$, the coalition structure $CS = (\{1+\varepsilon, 1\}, \{1+2\varepsilon\})$ is blocked by $\{1+2\varepsilon, 1\}$, $CS^{'} = (\{1+2\varepsilon, 1+\varepsilon\}, \{1\})$ is blocked by $\{1+\varepsilon, 1\}$, and $CS^{''} = (\{1+2\varepsilon, 1\}, \{1+\varepsilon\})$ is blocked by $\{1+2\varepsilon, 1+\varepsilon\}$.

Proposition 5 *Consider an academic game with different agents and indivisible budgets. Then a pairwise stable coalition structure can be found in polynomial time when it exists.*

Proof The game is an instance of the stable roommates problem with ties and incomplete lists, where an agent i finds another agent j unacceptable if i prefers working alone instead of forming a pair with j.